Bulk Resilience of Fibers

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Synopsis

A theory has been developed for the average bending deformation of fibers in an assembly subjected to compression. It is shown that the maximum average bending deformation of the fibers does not exceed 17°, implying that the method of bulk resilience cannot be used as an indicator of elastic recovery of fibers for any precision measurements.

INTRODUCTION

A considerable amount of experimental work[1–5] has been reported on compressibility and compressional resilience of fiber plugs. Theoretical approach to relate the deformation of individual fibers with the compression of the plug seems to have been made only by Van Wyk.[6] In developing a theory to obtain the compression of the fiber mass in terms of the applied load, Van Wyk considered the fiber as a rod supported at a large number of points spaced a distance $2b$ apart. The force F at the midpoint of each segment, assumed horizontal, was obtained as

$$F = 24iEy/b^3$$

where i is the moment of intertia of cross section of the segment, E is the Young's modulus of the material, and y the displacement of the midpoint of the segment. Since, however, in a fiber assembly, the segments do not remain horizontal but are randomly oriented, the above simplification does not appear to be quite admissible.

A study of bulk resilience of cotton was carried out recently in this laboratory with a view to examine whether this method could be used for assessing the efficacy of resin treatments usually meant for increased elastic recovery. If successful, it was proposed to adopt the method for identifying—at the fiber stage itself—cotton varieties that are more suited for resin finishing treatments. The method used for measurement of bulk resilience was that developed by Fox and Finzel.[7] A plug of cotton fibers weighing 1 g was contained in a metallic cylinder of 1 in. diameter mounted on the crosshead of the Instron Tensile Tester. This fiber mass was subjected to a number of compression–decompression cycles with a piston attached to the load cell of the Instron. Bulk-resilience measurement was made only after conditioning the sample in this manner.

The above experimental study showed that, although crosslinking treatment brought about an overall increase in bulk resilience, there was considerable overlap in the values of this property among samples treated to

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markedly different levels of crosslinking and hence of tensile elasticity. The results also failed to show any measurable difference among different varieties of cotton. This relative insensitivity of bulk resilience to structural changes has been explained in this paper by an analytical approach whereby it has been shown that, in the bulk test, individual fiber deformation is extremely small.

THEORY

Let us consider an assembly of fibers enclosed in a cylinder. This assembly is assumed to be small enough in size so that the weight of upper portions of the assembly does not compress the lower portions to any significant extent. In this assembly, each fiber is bent at a number of places where it comes in contact with other fibers. A segment of fiber between two adjacent contact points has a very large radius of curvature, so that it can be assumed to be straight. We call this a "fiber element." As has been shown by Duckett and Cheng[8] and Komori and Makishima,[9] the average length of the fiber element for random distribution of fibers is close to the value calculated by Van Wyk, namely, about 15 times the diameter for cotton fibers under conditions in which bulk resilience measurements are generally carried out.

It has been shown by Stearn[10] that when such a mass of fibers is compressed to half the original size, the number of contact points increases only by a factor of about 1.2. Hence, in the limits in which the experiments for bulk recovery are usually carried out, i.e., the extent of compression is less than half the original volume, it can be assumed that the element length would effectively remain the same during compression in the course of the experiment.

In the assembly of fibers there are a large number of elements like this. Since there is no preferential direction of alignment for the fiber elements, we assume that the fiber elements are randomly oriented.

In short, the above assumptions can be stated as: (i) fibers lie in a zigzag manner in the assembly, each fiber bending at points where it comes in contact with other fibers; (ii) the fiber segments between two successive bends (referred to as fiber elements) are essentially straight and of the same length a; (iii) the fiber elements are randomly oriented.

Let us fix rectangular coordinates in the fiber mass with z-axis along the vertical. Let θ (range 0–$\pi/2$) be the angle between a fiber element and the z-axis and ϕ (range 0–2π) be the angle between the x-axis and the projection of the element on the horizontal x–y plane.

If all the elements are imagined to be at the origin, the number of elements between θ and $\theta + d\theta$ will depend on the area of the annular ring formed on the surface of the hemisphere of radius a (Fig. 1). Hence, the probability P that an element lies between θ and $\theta + d\theta$ is

$$P = \frac{2\pi a^2 \sin\theta \, d\theta}{2\pi a^2} = \sin\theta \, d\theta$$

Since ϕ can take any value between 0 and 2π on the x–y plane, the prob-

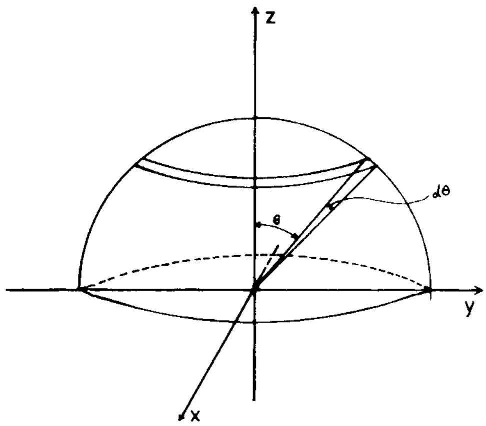

Fig. 1. Sketch showing probability of an element lying between (θ) and ($\theta + d\theta$).

ability that an element has inclination between ϕ and $\phi + d\phi$ is $d\phi/2\pi$. Thus, probability P′ that an element lies between (θ and $\theta + d\theta$) and (ϕ and $\phi + d\phi$) is

$$P' = \sin\theta\, d\theta \frac{d\phi}{2\pi} \tag{1}$$

Now, let us consider a case of two successive elements specified by angles (θ_1, ϕ_1) and (θ_2, ϕ_2) belonging to the same fiber. The angle between these two elements can be obtained from their direction cosines (l, m, n) which are

$$l_1 = \sin\theta_1\cos\phi_1, \quad m_1 = \sin\theta_1\sin\phi_1, \quad n_1 = \cos\theta_1$$
$$l_2 = \sin\theta_2\cos\phi_2, \quad m_2 = \sin\theta_2\sin\phi_2, \quad n_2 = \cos\theta_2$$

If ψ is the angle between the elements, then ($\pi - \psi$) is the angle made by lines having the above direction cosines. Thus,

$$\begin{aligned}\cos(\pi - \psi) &= l_1 l_2 + m_1 m_2 + n_1 n_2\\ &= \sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2\\ &\quad + \sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2 + \cos\theta_1\cos\theta_2\end{aligned} \tag{2}$$

To find the average value of ψ, from eqs. (1) and (2),

$$\overline{(\pi - \psi)} = \int\!\!\int\!\!\int\!\!\int [\cos^{-1}(\sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2 + \sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2 + \cos\theta_1\cos\theta_2)]\sin\theta_1\sin\theta_2\, d\theta_1\, d\theta_2 \frac{d\phi_1\, d\phi_2}{4\pi^2} \tag{3}$$

where the limits of integration are

for θ_1 and θ_2, from 0 to $\pi/2$ for ϕ_1 and ϕ_2, from 0 to 2π

The integral can be evaluated numerically by Simpson's one-third and Weddler's rules[11] for double integration. This gives

$$\overline{(\pi - \psi)} = 72.8°$$

or

$$\bar{\psi} = 107.2° \tag{4}$$

Now, the probability of an element making an angle between θ and $\theta + d\theta$ is $\sin\theta\, d\theta$. Therefore, average θ for elements in the assembly is

$$\bar{\theta} = \int_0^{\pi/2} \theta \sin\theta\, d\theta = 1^c = 57.3° \tag{5}$$

We consider a model of bulk of fibers in which all the elements make an angle of 1^c with the direction of compression (assumed to be vertical), and, also, consecutive elements on the same fiber make an angle of 107.2° with each other. This is an idealized model of a bulk of fibers which has random orientation of the elements. We call this an averaged model of a mass of fibers. This means that

$$\theta_1 = 57.3° = \theta_2 \quad \text{and} \quad \psi = 107.2°$$

Now, we put the x, y-axes in such a way that $\phi_1 = 0$ for an element. We need to find ϕ_2. From eq. (2), substituting proper values, we get

$$\cos 72.8° = \sin^2 57.3° \cos\phi_2 + \cos^2 57.3° \tag{6}$$

Therefore,

$$\cos\phi_2 = 0.0055$$

or

$$\phi_2 = 89°41' \tag{7}$$

This means that $\phi_2 - \phi_1 = 89°41'$ or $\Delta\phi$ between two consecutive elements on an average is 89°41′.

In bulk compression, this averaged assembly is compressed by the height factor f as compared to its original height. Assuming that all the elements are displaced in the same manner everywhere throughout the mass, all the elements now make angle θ' instead of 57.3° with the z-axis.

Projection of an element in the vertical direction is equal to $a \cos\theta'$. But the original projection of an element in the z-direction before compression is $a \cos 57.3°$. Since compression is to f times its original height, we must have

$$a \cos\theta' = f \times a \cos 57.3°$$

or

$$\cos \theta' = f \cos 57.3^\circ$$

We assume that $\Delta\phi$ (= 89°41′) does not change during compression of the fiber mass. In fact, projection of an element on x–y plane is sin 1^c (= 0.85) times the element length. This can at best become equal to the element length, when the fiber mass is hypothetically compressed to the limit of zero volume. This means that lateral shift will be always far less than 0.15 of the element length in the operating conditions. Therefore, lateral shift can be neglected:

$$\sin \theta' = (1 - f^2 \cos^2 57.3^\circ)^{1/2} \tag{8}$$

If ψ' is the angle between two consecutive elements in the compressed state, then, using eq. (2), we obtain

$$\cos(\pi - \psi') = (1 - f^2 \cos^2 57.3^\circ) \cos(\phi_1 - \phi_2) + f^2 \cos^2 57.3^\circ \tag{9}$$

This means that if a mass of fibers is compressed to f times its original height, then average value ψ' of the angle between two consecutive elements of the fiber is given by eq. (9). Knowing $\overline{\psi}$ (107.2°) from eq. (4), the deformation or the difference $(\overline{\psi} - \psi')$ can be easily worked out.

In the trival case, when $f = 1$,

$$\cos(\pi - \psi') = 0.2957$$

or

$$(\pi - \psi') = 72.8^\circ$$

or

$$\psi' = 107.2^\circ \tag{10}$$

which is also obtained from eq. (4).

In the extreme case, when the fiber mass is subjected to compression so that $f = 0$, we get

$$\cos(\pi - \psi') = \cos 89^\circ 41'$$

or

$$\pi - \psi' = 89^\circ 41' \tag{11}$$

Therefore,

$$\text{deformation} = \overline{\psi} - \psi' \simeq 17^\circ \tag{12}$$

The maximum deformation that can be caused is 17°. In the actual experiments the deformation will be quite less than 17°.

(Here, of course, when $f \to 0$, the deformation is not restricted to bending alone, but will start causing a lateral compression of the individual fibers at contact points).

CONCLUSION

In a bulk resilience experiment, the maximum deformation, i.e., the extent of bending of consecutive elements, is on an average less than 17°. This level of deformation is too small to differentiate samples having somewhat close values of bending recovery. When a set of such samples is tested for bulk resilience, the results will overlap considerably, signifying the lack of sensitiveness of bulk resilience to moderate changes in bending recovery. However, while testing different types of fibers such as cotton, wool, etc., which are structurally different and hence have widely different recovery properties, discernible differences in bulk resilience can be found to occur.

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NOMENCLATURE

a	length of fiber element in the fiber assembly
f	height of the fiber plug as a fraction of the height before compression
l,m,n	direction cosines of a fiber element
P	probability that a fiber element lies between θ and $\theta + d\theta$
P'	probability that a fiber element lies between both θ and $\theta + d\theta$ and ϕ and $\phi + d\phi$
θ	angle between a fiber element and the z-axis before the plug is compressed
θ'	angle between a fiber element and the z-axis after the plug is compressed
ϕ	angle between the x-axis and the projection of the fiber element on the x–y plane
ψ	angle between consecutive fiber elements before compression of the fiber plug
ψ'	angle between consecutive fiber elements in the compressed state of the plug

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